MOTION IMAGERY STANDARDS BOARD

RECOMMENDED PRACTICE

Generalized Transformation Parameters

MISB RP 1202

24 October 2013

1 Scope

This Recommended Practice (RP) describes a generalized method of transforming two-dimensional data (or points) from one coordinate system into a second two-dimensional coordinate system. This Generalized Transformation may be used for various image-to-image transformations such as an affine transformation by simply equating some parameters to be equal to zero. In addition, this Generalized Transformation may describe some homographic-like transformations.

This RP defines three items:

1) The different methods of implementation and constraints that need to be enforced to maintain certain transformation relationships.
2) The mandatory method of uncertainty propagation to be implemented on systems where uncertainty information is needed.
3) The KLV Local Data Set (LDS) that represents all the parameters for the Generalized Transformation.

2 References

2.1 Normative Reference

The following references and the references contained therein are normative.

[1] Edward M. Mikhail, James S. Bethel, and J. Chris McGlone. Introduction to Modern Photogrammetry. New York: John Wiley & Sons, Inc., 2001
[2] MISB ST 0107.1, Bit and Byte Order for Metadata in Motion Imagery Files and Streams, Jun 2011
[3] MISB RP 1010, Generalized Standard Deviation and Correlation Coefficient Metadata, Oct 2013

2.2 Informative References

[4] MISB EG 1002, Range Image Metadata Set, Aug 2010
[5] NGA.STND.0017_3.0, Community Sensor Model (CSM) Technical Requirements Document (TRD) Version 3.0, Nov 2010
[6] MISB RP 1107, Metric Geopositioning Metadata Set, Oct 2013

3 Revision History

Revision	Date	Summary of Changes
Original Draft	1/10/2012	• Original Draft of EG 1202 for review and comment
	5/28/2013	• Completion of Original Draft • Addition of Standard Deviation-Correlation Coefficient Floating Length Pack
	9/10/2013	• Revisions to original draft • Upgraded to Recommended Practice

4 Abbreviations and Acronyms

CCFPA	Combined Composite Focal Plane Array
CPT	Child-Parent Transformation
CSM TRD	Community Sensor Model Technical Requirements Documents
CT	Chipping Transformation
DPIT	Default Pixel-Space to Image-Space Transformation
EG	Engineering Guideline
FLOAT	IEEE Single precision floating point number
FLP	Floating Length Pack
FPA	Focal Plane Array
INT	IEEE Integer
KLV	Key-Length-Value
LDS	Local Data Set
MISB	Motion Imagery Standards Board
NDT	No Defined Transformation
OT	Optical Transformation
RP	Recommended Practice

5 Introduction

This RP defines a Generalized Transformation based on the foundational two-dimensional projective transformation. From the Generalized Transformation, this RP defines four types of commonly used derived transformations, and a methodology for extending support for additional derived transformations. All derived transformations assume that when a parameter is not given, it is equal to zero. This assumption helps in the implementation of transformations. As such, if all parameters are assumed to be equal to zero, then the resulting transformation returns an output identically equivalent to its input.

In addition, transformation data may be accompanied by uncertainty information that describes the quality of the transformation; however, it is not required. This RP defines a method to describe the standard deviation and correlation coefficient information that accompanies the transformation. To prevent incorrect error propagation, all constraints that describe the individual transformation must be accounted for when invoking the stochastic model.

Finally, a LDS (Local Data Set) is defined that contains the transformation parameters necessary to implement the Generalized Transformation. This LDS maps 16-byte Universal Keys assigned to each individual parameter of the Generalized Transformation to 1-byte Tags for efficiency purposes.

The transformation data provides the parameters for mapping between two-dimensional spaces. In order to use this transformation, it must be associated with one of the two-dimensional spaces, which provides the context of the transformation. In order to provide this context this LDS must be used within a "parent" KLV set. This means that this LDS is never used standalone or without being embedded in another LDS.

6 Generalized Transformation

The Generalized Transformation describes a class of two-dimensional projective transformations intended for image-space coordinate transformations. The two-dimensional projective transformation is the foundation of the Generalized Transformation. The purpose of this transformation is to define a mathematical mapping from points on one plane to points on another plane. For this reason, a system of homogeneous coordinates is used. The following two equations provide a mathematical description of the plane-to-plane projective transformation relation of input to output image coordinates.

$$x_{out} = \frac{(1-A)x_{in} + By_{in} + C}{Gx_{in} + Hy_{in} + 1} \qquad \textbf{Equation 1}$$

$$y_{out} = \frac{Dx_{in} + (1-E)y_{in} + F}{Gx_{in} + Hy_{in} + 1} \qquad \textbf{Equation 2}$$

The form of Equation 1 and Equation 2 is slightly different than how a projective transformation is typically described, where the terms (1 - A) and (1 - E) are normally expressed as just A and E respectively. This modification allows for all values in the expression to be equal to zero without any need for special cases. With all constants (A through H) equal to zero, the transformation yields coordinates identically equal to the input. This is advantageous because the transformation can always be executed regardless of the input data (e.g. if one or more of the parameters are zero).

As this transformation is a projective transformation, the inverse may be written as a function of the original parameters. This, again, has advantages because only one set of parameters is needed to define the forward transformation ***and*** the inverse transformation. The inverse of Equation 1 and Equation 2 derived through a series of algebraic steps results in Equation 3 and Equation 4 respectively.

$$x_{in} = \frac{\big((1-E) - FH\big)x_{out} + (CH - B)y_{out} + \big(BF - C(1-E)\big)}{\big(DH - G(1-E)\big)x_{out} + \big(GB - H(1-A)\big)y_{out} + \big((1-A)(1-E) - DB\big)} \qquad \textbf{Equation 3}$$

$$y_{in} = \frac{(GF - D)x_{out} + \big((1-A) - CG\big)y_{out} + \big(DC - F(1-A)\big)}{\big(DH - G(1-E)\big)x_{out} + \big(GB - H(1-A)\big)y_{out} + \big((1-A)(1-E) - DB\big)} \qquad \textbf{Equation 4}$$

Equations 1-4 define a number of two-dimensional projective transformations and their inverses. In many imagery applications, a two-dimensional affine transformation requires six parameters. This set of six parameters is a subset of the original eight parameters described above. The Appendix contains various formulations of projective transformations and the constraints needed to create various "standard" transformations.

6.1 Transformation Types

The derived transformations defined in this RP are identified in Table 1 and can be represented by their enumeration value in the subsequent Generalized Transformation Local Data Set (LDS).

Table 1: Derived Transformations List

Enumeration Value	Description	Units
0	Other – No Defined Transformation (NDT)	None
1	Chipping Transformation (CT)	Pixels
2	Child-Parent Transformation (CPT)	Millimeters
3	Default Pixel-Space to Image-Space Transformation (DPIT)	Millimeters
4	Optical Transformation (OT)	Millimeters

6.1.1 Other – No Defined Transformation

An enumeration value equal to "0" implies the transformation type is not defined; however, this does not prevent the user from exploiting the information contained within the Generalized Transformation LDS.

6.1.2 Chipping Transformation

An enumeration value equal to one "1" signifies the transmitted image is a chip (or sub-region) from a larger image. Examples of a chipped image are: 1) a sub-region of an image that may be digitally enlarged (zoom); 2) a sub-region of an image selected to reduce bandwidth, or to provide higher quality within the sub-region. Further information on this transformation is given in Section 7.1.1.

6.1.3 Child-Parent Transformation

An enumeration value equal to two "2" indicates the transformation of a child focal plane array (FPA) to its parent FPA (*e.g.* example defined in MISB EG 1002[4]). This CPT is a plane-to-plane transformation used to transform between FPA's in image space. Further description of this transformation is given in Section 7.1.2.

6.1.4 Default Pixel-Space to Image-Space Transformation

An enumeration value equal to three “3” is the default pixel-space to image-space transformation. Further information on this transformation is given in Section 7.1.3.

6.1.5 Optical Transformation

An enumeration value equal to four “4” indicates the pixel data of an image is a subset of an entire optical focal plane. This may occur in some Combined Composite Focal Plane Array (CCFPA) sensors, where multiple focal plane array detectors combine to image a single optical focal plane. This optical transformation is a plane-to-plane transformation to transform from FPA to the full optical image plane. Further description if this transformation is given in Section 7.1.4.

6.1.6 Extensibility for New Transformations

Additional derived transformations may be added to this RP to support new capabilities.

Requirement	
RP 1202-01	Additional derived transformations supported by the MISB shall be added to MISB RP 1202 Table 1 along with supporting information regarding type and use.

6.2 Uncertainty Propagation

In many applications, the knowledge of the uncertainty of all estimated values is critical to understand the performance of a system. Thus, it is desirable to provide a means to propagate the uncertainty information of the transformation parameters. The Generalized Transformation LDS utilizes the format described in MISB RP 1010[3] for transmitting the standard deviation and correlation coefficient information.

Requirement	
RP 1202-02	When uncertainty information of the Generalized Transformation parameters is available, uncertainty information shall be represented by a Standard Deviation Correlation Coefficient Floating Length Pack (FLP) in accordance with MISB RP 1010[3].

The Standard Deviation Correlation Coefficient FLP, as defined in MISB RP 1010, requires the parent LDS (*e.g.* the Generalized Transformation LDS in this case) to define the order of parameters to associate uncertainty information.

Requirement	
RP 1202-03	The matrix size in the Standard Deviation Correlation Coefficient FLP shall be eight (8) to represent all the parameters in the Generalized Transformation.

RP 1202-04	The Standard Deviation Correlation Coefficient FLP LDS shall order its entries for the eight elements of the Generalized Transformation LDS in the same order as the first eight parameters of MISP RP 1202 - Table 2.

The projective transformation is the general case of the two-dimensional to two-dimensional transformation and no constraints exist on the uncertainty propagation. Further information on how to handle the uncertainty propagation for other transformations is addressed in the Appendix.

6.3 Concatenation of Transformations

A benefit of projective transformations is that a combination of projective transformations is itself a projective transformation; however, the order in which these transformations are performed is critical. In the case of determining these transformations for sensor modeling purposes, which assumes an image-to-ground sequence, the order is defined by the following.

Requirement	
RP 1202-05	Transformations shall be performed in the following order: 1) chipping, 2) child-parent, 3) default pixel-space to image-space and 4) image-space coordinates imaged on the focal plane into the full optical image-space coordinate system.

- The chipping or digital zoom transformation is the first transformation to be performed. This transformation transforms the image coordinates of the chipped or digitally zoomed image into the original image coordinate system. This is the transformation described in Section 7.1.1.
- The Child-Parent transformation is the second transformation to be performed. This transformation transforms the original image coordinates above of the child image into the image coordinate system of a parent image. This is the transformation described in Section 7.1.2.
- The default pixel-space to image-space transformation is the third transformation to be performed. This transforms the pixel coordinates into units of millimeters and moves the origin to the center of the image. This is the transformation described in Section 7.1.3.
- The fourth and final transformation transforms the image-space coordinates imaged on the focal plane into the full optical image-space coordinate system. This is the transformation described in Section 7.1.4.

The ground-to-image projection sequence is the inverse of the image-to-ground sequence. Uncertainty information may accompany all of the above transformations.

6.4 Generalized Transformation Local Metadata Set

The Generalized Transformation LDS as defined in this RP has the following requirements:

Requirement	
RP 1202-06	All metadata shall be expressed in accordance with MISB ST 0107[2].
RP 1202-07	The version of MISB RP 1202 utilized shall always be sent in the Generalized Transformation LDS.
RP 1202-08	When the enumeration value corresponding to the transformation type is not populated in the Generalized Transformation LDS, the value shall be assumed to be equal to zero indicating No Defined Transformation.
RP 1202-09	The MISB RP 1202 Local Data Set shall be embedded within a LDS that provides context for the transformation.

Tags and Keys within the Generalized Transformation LDS Table 2 defines the Generalized Transformation LDS data elements and data order.

Table 2: Generalized Transformation LDS

Local Set Key						Name
06.0E.2B.34.02.0B.01.01.0E.01.03.05.05.00.00.00 (CRC 40498)						Generalized Transformation LDS
Constituent Elements						
Tag ID	**Key**	**Name**	**Symbol / Notes**	**Units / Range**	**Format**	**Length (bytes)**
1	06.0E.2B.34.01.01.01.01.0E.01.02.02.81.01.00.00 (CRC 39709)	x Equation Numerator-x factor	A in Equation 1	N/A	FLOAT	4
2	06.0E.2B.34.01.01.01.01.0E.01.02.02.81.02.00.00 (CRC 49741)	x Equation Numerator-y factor	B in Equation 1	N/A	FLOAT	4
3	06.0E.2B.34.01.01.01.01.0E.01.02.02.81.03.00.00 (CRC 62845)	x Equation Numerator-Constant factor	C in Equation 1	N/A	FLOAT	4
4	06.0E.2B.34.01.01.01.01.0E.01.02.02.81.04.00.00 (CRC 28909)	y Equation Numerator-x factor	D in Equation 2	N/A	FLOAT	4
5	06.0E.2B.34.01.01.01.01.0E.01.02.02.81.05.00.00 (CRC 18397)	y Equation Numerator-y factor	E in Equation 2	N/A	FLOAT	4
6	06.0E.2B.34.01.01.01.01.0E.01.02.02.81.06.00.00 (CRC 7821)	y Equation Numerator-Constant factor	F in Equation 2	N/A	FLOAT	4
7	06.0E.2B.34.01.01.01.01.0E.01.02.02.81.07.00.00 (CRC 10685)	Denominator-x factor	G in Equation 1 and Equation 2	N/A	FLOAT	4
8	06.0E.2B.34.01.01.01.01.	Denominator-y factor	H in Equation 1 and Equation 2	N/A	FLOAT	4

	0E.01.02.02.81.08.00.00 (CRC 1420)					
9	06.0E.2B.34.02.05.01.01. 0E.01.03.03.21.00.00.00 (CRC 64882)	Standard Deviation and Correlation Coefficient FLP	This Key defined in MISB RP 1010[3]	N/A	N/A	N/A
10	06.0E.2B.34.01.01.01.01. 0E.01.02.05.05.00.00.00 (CRC 56368)	Document Version	document_version	[0 – 255]	BER-OID	1
11	06.0E.2B.34.01.01.01.01. 0E.01.02.03.5F.00.00.00 (CRC 3109)	Transformation Enumeration	Transformation Type defined in Table 1	[0 – 255]	UINT8	1

7 Appendix

This appendix provides further details on the mapping of the parameters in the unique transformations represented within this RP. These transformation types, their inverses, and the uncertainty propagation are given.

As a general rule, the uncertainty propagation is defined as if all eight parameters are being used. With this assumption, special cases are not needed on the algorithm, or in usage. It is the responsibility of the data provider to populate the uncertainty information correctly in order to properly represent the uncertainties in the transformation.

7.1 Generalized Transformation LDS

The Generalized Transformation Local Data Set supports a number of transformation types that may be needed in the development of a sensor model. One transformation, the default pixel to image-space transformation (enumeration value = 3), is performed on ***all*** data. The remaining transformation types are performed according to the needs of the dataset; however, a specific ordering of these transformations is mandatory.

The following four subsections describe these transformations.

7.1.1 Chipping Transformation (CT)

The chipping transformation (enumeration value = 1) is utilized for image chipping and a special subset of image chipping known as digital zoom. The chipping transformation is performed in the pixel coordinate system defined by the Community Sensor Model (CSM) Technical Requirements Document (TRD)[5] (*e.g.* line and sample (or row and column) measured from the upper left hand corner). This is shown in Figure 1 and Figure 2.

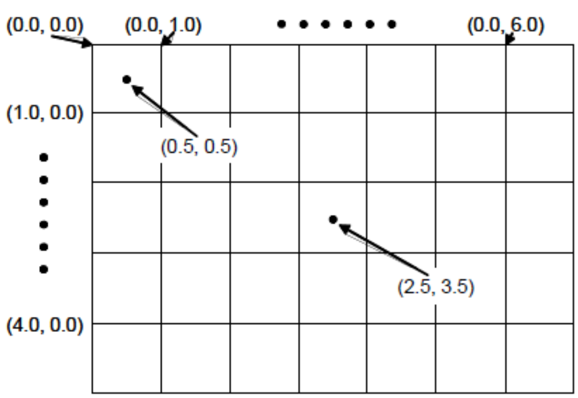

Figure 1: Pixel Coordinate System per CSM TRD

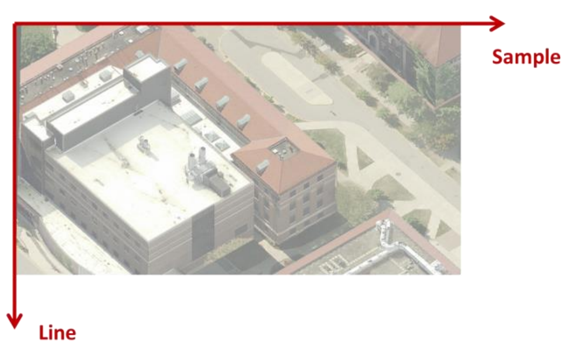

Figure 2: Pixel Coordinate System in an Image

The general form of the chipping transformation is given in Equation 5. A chipped or zoomed image is a sub-region of a larger image without rotation, as illustrated in Figure 3. The transformations needed for executing a sensor model must transform the chipped image coordinates into the original image coordinate space (*i.e.* the location of the original pixels must be known).

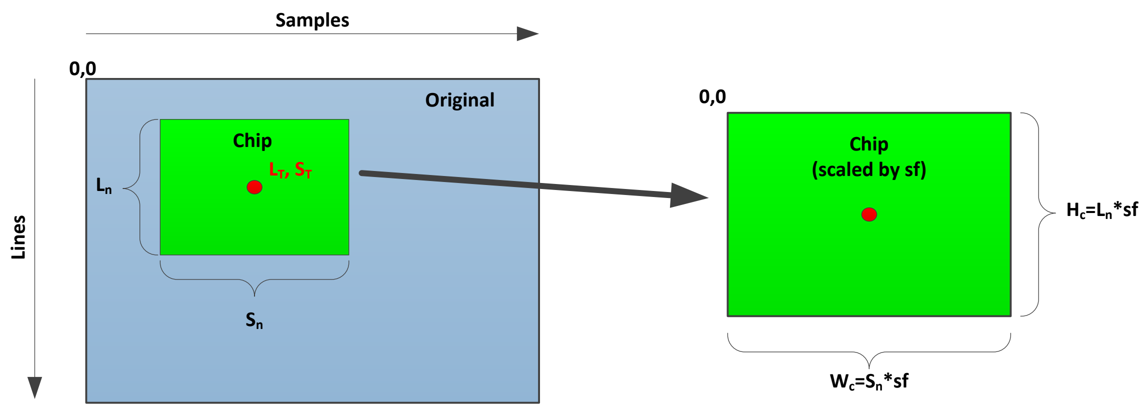

Figure 3: Example Chipping Transformation

$$\begin{bmatrix} L_O \\ S_O \end{bmatrix} = \begin{bmatrix} \frac{1}{sf} & 0 \\ 0 & \frac{1}{sf} \end{bmatrix} \begin{bmatrix} L_C \\ S_C \end{bmatrix} + \begin{bmatrix} L_T - \left(\frac{1}{sf}\right)\frac{H_C}{2} \\ S_T - \left(\frac{1}{sf}\right)\frac{W_C}{2} \end{bmatrix} \quad \textbf{Equation 5}$$

The transformation parameters for chipping are computed from a combination of a number of parameters, which are described below.

$$A = 1 - \frac{1}{sf} \quad \textbf{Equation 6}$$

$$B = 0 \quad \textbf{Equation 7}$$

$$C = L_T - (1 - A)\frac{H_C}{2} \quad \textbf{Equation 8}$$

$$D = 0 \quad \textbf{Equation 9}$$

$$E = 1 - \frac{1}{sf} \quad \textbf{Equation 10}$$

$$F = S_T - (1 - E)\frac{W_C}{2}$$ **Equation 11**

$$G = 0$$ **Equation 12**

$$H = 0$$ **Equation 13**

The translation values, L_T and S_T, in Equation 8 and Equation 11 describe the location of the center of the chipped image within the original image. The value sf is the scale factor used to scale the image. It is assumed that sf is equally applied to both a line and sample. The variables L and S describe the line and sample coordinates, respectively, of the point of interest. In Equation 5, the subscript O refers to the original image coordinates and the subscript C refers to the chipped image coordinates. Finally, the variables H_C and W_C are the chipped image height and width, respectively.

A special case of the chipping transformation is a Digital Zoom of the original image. A Digital Zoom uses the center region of the original image and produces a new image with new coordinates and same dimensions as the original image, as illustrated in Figure 4. For this special case the last terms of Equation 5 can be computed from the size of the original image as shown in Equation 14.

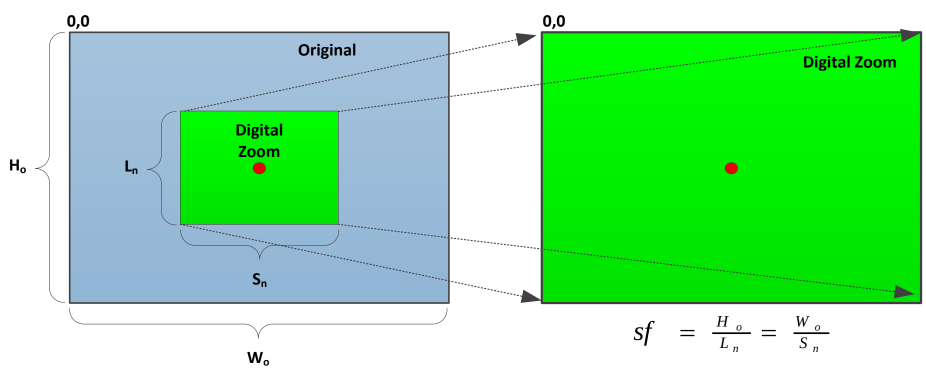

Figure 4: Digital Zoom Transformation

$$\begin{bmatrix} L_O \\ S_O \end{bmatrix} = \begin{bmatrix} \frac{1}{sf} & 0 \\ 0 & \frac{1}{sf} \end{bmatrix} \begin{bmatrix} L_C \\ S_C \end{bmatrix} + \begin{bmatrix} \frac{H_O}{2}\left(1 - \frac{1}{sf}\right) \\ \frac{W_O}{2}\left(1 - \frac{1}{sf}\right) \end{bmatrix}$$ **Equation 14**

The transformation parameters for digital zoom are computed from a combination of a number of parameters, which is described below.

$$A = 1 - \frac{1}{sf}$$ **Equation 15**

$$B = 0$$ **Equation 16**

$$C = \frac{H_O}{2}\left(1 - \frac{1}{sf}\right)$$ **Equation 17**

$$D = 0$$ **Equation 18**

$$E = 1 - \frac{1}{sf}$$ **Equation 19**

$$F = \frac{W_O}{2}\left(1 - \frac{1}{sf}\right)$$ **Equation 20**

$$G = 0$$ **Equation 21**

$$H = 0$$ **Equation 22**

The value sf is the scale value used to apply a digital zoom to an image. For example, for a 2X digital zoom sf = 2. It is assumed sf is equally applied to both line and sample. The variables L and S describe the line and sample coordinates, respectively, of the point of interest. The subscript O is in reference to the original image coordinates and the subscript C is in reference to the chipped image coordinates. Finally, the variables H_O and W_O are the original image height and width, respectively.

The chipping transformation only produces rescaled and translated images. The parameters that describe the chipping transformation are assumed to be known needing no uncertainty information about these parameters. Because of this, there is typically no stochastic model that accompanies this transformation.

The values defined in Equation 15 through Equation 22 or Equation 6 through Equation 13 can be used to define the inverse transformation using Equation 3 and Equation 4.

7.1.2 Child-Parent Transformation (CPT)

The Child-Parent Transformation (enumeration value = 2) is used in transforming a child focal plane array to its parent focal plane array. These two arrays are related within multiple sensors. An example of this is a co-boresighted sensor system with sensors contained within the same turret. In this formulation, one focal plane must be chosen as the "parent" focal plane. This focal plane is what metadata, such as photogrammetry metadata, is in reference. The "child" focal plane is the image being transformed into the "parent" sensor's coordinate system. The transformation can include rotation, translation and scaling. This is done by applying an eight parameter transformation via the General Transformation described in Equation 1 and Equation 2, where the child image coordinates are represented by the "*in*" subscripts, the parent image coordinates are represented by the "*out*" subscripts, and the variables L_{in} and S_{in} to describe the line and sample coordinates in the child image and the variables L_{out} and S_{out} to describe the line and sample coordinates in the parent image.

$$L_{out} = \frac{(1-A)L_{in} + BS_{in} + C}{GL_{in} + HS_{in} + 1} \qquad \textbf{Equation 23}$$

$$S_{out} = \frac{DL_{in} + (1-E)S_{in} + F}{GL_{in} + HS_{in} + 1} \qquad \textbf{Equation 24}$$

The CPT may be inserted into the parent LDS invoking this transformation. The CPT does not require any unique mapping into the metadata stream.

The transformation values in Equation 23 and Equation 24 can be used to define the inverse transformation in Equation 3 and Equation 4.

7.1.3 Default Pixel-Space to Image-Space Transformation (DPIT)

The default pixel-space to image-space transformation has two representations. The first is the CSM TRD defined approach for motion imagery; the second is a generalized approach for other imagery modalities. The definitions for these cases are in 7.1.3.1 and 7.1.3.2 respectively.

7.1.3.1 CSM TRD Default Pixel to Image-Space Transformation

For motion imagery, the default transformation (enumeration value = 3) is the assumed transformation in constructing a CSM compliant sensor model of a ***full*** image. That is, the full focal pane array is transmitted and represented by the metadata stream. This is the transformation assumed to be contained within MISB RP 1107[6] that converts the pixel coordinates into the image coordinates for the sensor model. This transformation is defined by Equation 25.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & p2m_x \\ -p2m_y & 0 \end{bmatrix} \begin{bmatrix} L - \frac{H}{2} \\ S - \frac{W}{2} \end{bmatrix} \qquad \textbf{Equation 25}$$

The value $p2m_x$ and $p2m_y$ are the dimensions given to each individual pixel. These pixels may, or may not, be square. The variables L and S describe the line and sample pixel coordinates, respectively, of the point of interest. The variables H and W are the full image height and width, respectively. Finally, the variables x and y are the image coordinates.

This transformation may be inserted into the parent LDS invoking the default transformation; however, it is not needed because it is assumed to be contained within MISB RP 1107. If it is present in the metadata stream, the following mapping is applied:

$A = 1$	**Equation 26**
$B = p2m_x$	**Equation 27**
$C = -p2m_x \dfrac{W}{2}$	**Equation 28**
$D = -p2m_y$	**Equation 29**
$E = 1$	**Equation 30**
$F = p2m_y \dfrac{H}{2}$	**Equation 31**
$G = 0$	**Equation 32**
$H = 0$	**Equation 33**

As this is the default transformation and considered a known constant, there is typically not a stochastic model that accompanies this transformation.

The values defined in Equation 25 through Equation 33 may be used to define the inverse transformation using Equation 3 and Equation 4.

7.1.3.2 Generalized Pixel to Image-Space Transformation

For other imagery modalities not using the CSM TRD defined pixel-space to image-space transformation, the transformation (enumeration value = 3) is the assumed generalized transformation that defines the transformation between the pixel image coordinate system and the image coordinate system. This is accomplished by applying the Generalized Transformation described in Equation 34 and Equation 35, where the pixel image coordinates are represented by the "*in*" subscripts, and the image coordinates are represented by the "*out*" subscripts. This transformation was defined in the main body of text, but is being repeated below for reference.

$$x_{out} = \frac{(1 - A)L_{in} + BS_{in} + C}{GL_{in} + HS_{in} + 1} \quad \textbf{Equation 34}$$

$$y_{out} = \frac{DL_{in} + (1 - E)S_{in} + F}{GL_{in} + HS_{in} + 1} \quad \textbf{Equation 35}$$

The generalized default pixel-space to image-space transformation introduces no new variables. It uses the variables L_{in} and S_{in} to describe the line and sample coordinates in the pixel image and uses the variables x_{out} and y_{out} to describe the x and y coordinates in the image coordinate system. The transformation values in Equation 34 and Equation 35 may be used to define the inverse transformation in Equation 3 and Equation 4.

7.1.4 Optical Transformation (OT)

The optical transformation (enumeration value = 4) is the assumed transformation used in transforming a FPA to a CCFPA. These two arrays are optically related within one sensor. The relationship occurs when the pixel data of the image is a subset of the entire optical focal plane. A transformation must be done to transform the focal plane array to its composite focal plane array. Similar to the CPT, the two arrays consist of an *"out"* and *"in"* array. The CCFPA is considered the *"out"* array while the originating FPA is considered the *"in"* array. The eight parameter transformation via the General Transformation described in Equation 1 and Equation 2 is applied, and is repeated below for reference.

$$x_{out} = \frac{(1-A)x_{in} + By_{in} + C}{Gx_{in} + Hy_{in} + 1}$$ **Equation 36**

$$y_{out} = \frac{Dx_{in} + (1-E)y_{in} + F}{Gx_{in} + Hy_{in} + 1}$$ **Equation 37**

The OT introduces no new variables. It uses the variables x_{in} and y_{in} to describe the originating FPA image coordinates, and the variables x_{out} and y_{out} to describe the CCFPA image coordinates.

The OT may be inserted into the parent LDS invoking the optical transformation. The OT does not require any unique mapping into the metadata stream.

The OT above is the General Transformation described in the main body of this document. Uncertainty propagation for the OT is the same as the General Transformation. This was described in depth in section 6.2.

The values defined in Equation 36 and Equation 37 may be used to define the inverse transformation in Equation 3 and Equation 4.